J. Phys. B: At. Mol. Opt. Phys. **43** (2010) 105502 (5pp) doi:10.1088/0953-4075/43/10/105502

Photon-energy qubit generation by spontaneous emission in a V-type system

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Received 21 December 2009, in final form 12 April 2010
Published 5 May 2010
Online at stacks.iop.org/JPhysB/43/105502

Abstract
We propose a scheme for generating arbitrary pure-sate photon-energy qubits by adiabatic manipulations of a cavity-embedded v-type level system. The dynamics of the system is analyzed as well as the effect of various parameters including decoherence. Our calculations indicate that an equal coupling constant v-type system is the preferable choice. The realization of the proposed scheme is possible in Rb-based implementations—aimed at applications in fiber-optical quantum communication.

(Some figures in this article are in colour only in the electronic version)

There is constantly increasing interest in quantum information processing in general and in quantum communications in particular. Different methods have been proposed to tackle the challenge of transporting pure-state qubits over relatively long distances. One of the promising qubit implementations is based on photon polarization, combining simple generation and manipulation with long coherence times in free-space propagation. However, the use of photon polarization in the commercially available infrastructure of fiber optics is problematic due to the stress-induced random polarization changes [1]. Photons with different energy components comprising an energy qubit [2, 3] can have negligible state mixing during the propagation along the fiber with small nonlinearities and deterministic dispersion characteristics.

We propose a method for generating arbitrary photon-energy qubits with desired phase and amplitude relations, using adiabatic manipulations of a V-type atom–photon system within a cavity, where the generated photon can be switched out of the cavity on demand. We show that the four-level system depicted on figure 1 generates energy qubits:

$$\Psi = \alpha\left|1_{\omega_1}0_{\omega_2}\right\rangle + \beta\left|0_{\omega_1}1_{\omega_2}\right\rangle \tag{1}$$

where α and β are arbitrary complex numbers, satisfying normalization condition, ω_1 and ω_2 the photon frequencies, determined by $|1\rangle - |2\rangle$ and $|3\rangle - |2\rangle$ energy-level separation.

The concept of the system's operation is as follows: initially, the electron occupies the ground state $|2\rangle$. It is then pumped by a π-pulse to the excited state $|3\rangle$ [4] (figure 1(a)) and subsequently 'spread' between levels $|1\rangle$ and $|3\rangle$ using partial stimulated Raman adiabatic passage (STIRAP) technique, assisted by an auxiliary level $|4\rangle$ [5] (figure 1(b)). Finally, the electron decays to the ground state, generating a photon-energy qubit as described in equation (1) (figure 1(c)). The STIRAP technique provides a method to control the relative amplitudes of the qubit energy components, while the relative phase can be easily introduced by an additional dispersive element. In general STIRAP uses an auxiliary state $|4\rangle$ for an adiabatic transfer between level $|1\rangle$ and $|3\rangle$. In principle, the following sequence is also possible: a π-pulse to transfer the population from $|2\rangle$ to $|3\rangle$, and sequence of pulses—$\omega_{12} \stackrel{\Delta}{=} \omega_1$ overlapped by $\omega_{32} \stackrel{\Delta}{=} \omega_2$ in this scheme, so that level $|2\rangle$ is used twice. However, level $|3\rangle$ itself has a finite lifetime, and hence, the use of level $|2\rangle$ for STIRAP will be accompanied by spontaneous emission from $|3\rangle$ to $|2\rangle$. On the other hand, an additional level $|4\rangle$ allows applying the $|3\rangle$ to $|2\rangle$ π-pulse simultaneously with the $|1\rangle$–$|4\rangle$ coherence buildup, thus circumventing the spontaneous emission issue. In a STIRAP process the auxiliary level $|4\rangle$ is never populated; therefore, it should not induce electron decoherence or escape from Hilbert space. In our specific scheme, level $|4\rangle$ is situated at a higher energy than the other levels, so that no spontaneous transitions to level $|4\rangle$ are possible, whereas stimulated transitions to level $|4\rangle$ cannot occur under the rotating wave approximation. Therefore, level $|4\rangle$ should not induce photon decoherence either.

By employing the partial STIRAP technique, the electron–photon system can be prepared in the following state, where α' and β' are coefficients that can be arbitrarily set by

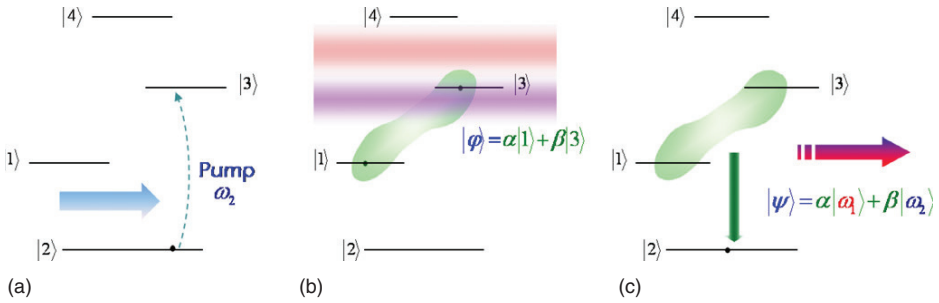

Figure 1. The proposed energy qubit generation process. (a) A π pulse excites the electron from the ground level $|2\rangle$ to state $|3\rangle$. (b) Using two lasers and an additional upper level $|4\rangle$ a partial STIRAP process is performed on the electron. (c) The electron that is now in a coherent superposition between states $|1\rangle$ and $|3\rangle$ decays to state $|2\rangle$, emitting a photon in coherent superposition of energies.

using proper parameters of the STIRAP process:

$$|\psi_{\text{system}}\rangle = (\alpha'|1\rangle + \beta'|3\rangle) \otimes \left|0_{\omega_1} 0_{\omega_2}\right\rangle \tag{2}$$

where $\left|0_{\omega_1} 0_{\omega_2}\right\rangle$ indicates the photon vacuum state.

The following Jaynes–Cummings Hamiltonian [6] describes the dynamics of the system:

$$H_{\text{JC}} = g_1 \cdot \left(\hat{a}_1^{\dagger}|2\rangle\langle 1| + \hat{a}_1|1\rangle\langle 2|\right) + g_3 \cdot \left(\hat{a}_2^{\dagger}|2\rangle\langle 3| + \hat{a}_2|3\rangle\langle 2|\right) \tag{3}$$

where $\hat{a}^{\dagger}$ and $\hat{a}$ are the respective photon creation and annihilation operators and $g_{1,3}$ are the coupling constants of the corresponding transitions (figure 1). The evolution of the initial state (equation (2)) according to the Hamiltonian will result in the generation of the desired photon state at a discrete set of times $t_k = \frac{h}{4\cdot|g_1|} \cdot (2 \cdot k + 1)$ where k is a natural number and h is Planck's constant. The dynamics of the system is determined by two oscillation frequencies, and consequently k has to fulfil an additional condition: $2 \cdot m + 1 = |g_3/g_1| \cdot (2 \cdot k + 1)$, where m is a natural number. At these times t_k the atom–photon system is described by

$$|\psi_{\text{final}}\rangle = |2\rangle \otimes \left(\alpha'\widetilde{g}_1\left|1_{\omega_1} 0_{\omega_2}\right\rangle + \beta'\widetilde{g}_3\left|0_{\omega_1} 1_{\omega_2}\right\rangle\right) \tag{4}$$

where $\widetilde{g}_1 = g_1/|g_1|, \widetilde{g}_3 = g_3/|g_3|$ are the phases associated with the transition coupling coefficients.

The appropriate 9×9 atom–photon density matrix $\rho = |\psi_{\text{final}}\rangle\langle\psi_{\text{final}}|$, where the electron is described in a $|1\rangle, |2\rangle, |3\rangle$ space and the photon in the $\left|0_{\omega_1} 0_{\omega_2}\right\rangle, \left|1_{\omega_1} 0_{\omega_2}\right\rangle, \left|0_{\omega_1} 1_{\omega_2}\right\rangle$ space is block-diagonal at initial and final times $\rho = \rho_{\text{at}} \otimes \rho_{\text{phot}}$, with the atom reduced matrix in the final state being simply the ground state:

$$\rho_{\text{at}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

and the reduced density matrix of the photon in the final state is

$$\rho_{\text{phot}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & |\alpha'\widetilde{g}_1|^2 & \alpha'\widetilde{g}_1(\beta'\widetilde{g}_3)^* \\ 0 & (\alpha'\widetilde{g}_1)^*\beta'\widetilde{g}_3 & |\beta'\widetilde{g}_3|^2 \end{pmatrix} \tag{6}$$

with $|\widetilde{g}_1|^2 = |\widetilde{g}_3|^2 = 1$, corresponding to the pure photon state

$$\Psi = \alpha\left|1_{\omega_1} 0_{\omega_2}\right\rangle + \beta\left|0_{\omega_1} 1_{\omega_2}\right\rangle \tag{7}$$

by defining $\alpha \triangleq \alpha'\widetilde{g}_1, \beta \triangleq \beta'\widetilde{g}_3$.

Only the magnitude of the state amplitudes is determined by the preparation, while the relative phase is fixed by the coupling constants. To allow an arbitrary selection of the relative phase between ω_1 and ω_2 components of a photon, a dispersive element can be used after the photon is emitted.

The closed atom–photon system exhibits Rabi oscillations interchanging the energy between the atom and the field. However, using a switchable mirror, it is possible in principle to switch-out photons from the cavity at the desired time. This allows the switch-open timing to be the same for any amplitude relation of the initial electron state.

The evolution of the system for various coupling constants was calculated using the Hamiltonian of equation (3) (figures 2 and 3). The off-diagonal elements of the photon part of the density matrix (figures 2 and 3(a)) are related to the dynamics of the amplitudes of the two-photon frequency components, allowing us to determine a desired qubit state. The probability of a photon-energy qubit to be emitted is given by population probability of the atomic ground state $|2\rangle$ (figures 2 and 3(c)), the probability of measuring the emitted photon with one of the frequency components is given by the diagonal elements (figures 2 and 3(b)), while the purity of the photon state is given by the off-diagonal elements of ρ_{phot} (figures 2 and 3(a)). In the special case of equal coupling constants (figure 2), the two transitions have equal Rabi frequencies. One advantage of this situation is that in every Rabi cycle the electron populates its ground state with unity probability, in contrast to the different-coupling-constant case (figure 3) where the more complex dynamics makes these events much less frequent. Moreover, for equal coupling, measurement of the electron in the ground state (at any time in the Rabi cycle) results in the photon exactly in the desired superposition state, which is not the case for the different coupling scenario—improper timing will result in a different qubit than desired. The proposed system is feasible with different strengths; however, it is more easily implemented if equal strength can be achieved. It stems from the fact that for equal coupling, the probability of finding the electron in the ground state is path independent at all times, making this configuration flexible.

For evaluating practical realizations, we consider the effect of inaccuracies in the photon extraction time (mirror switch-off time). The time dependence of the reduced photon density matrix is

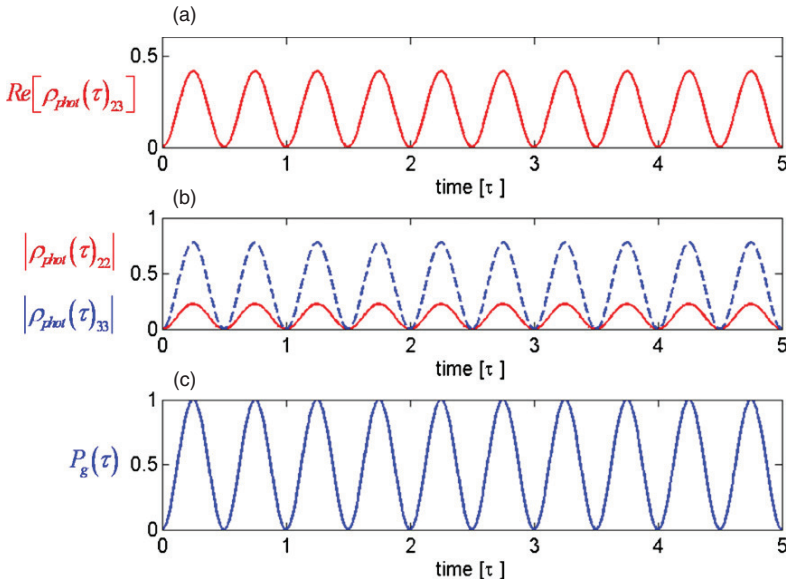

Figure 2. Time evolution of the photon part of the density matrix for *equal* coupling strength of the transition paths, with time in units of $\tau = g_1/h$. (a) Real (solid line) and imaginary (dashed line) parts of the off-diagonal element. (b) Absolute values of the diagonal elements for the ω_1 path (solid line) and the ω_2 path (dashed line). (c) Probability for the population of the atomic ground state.

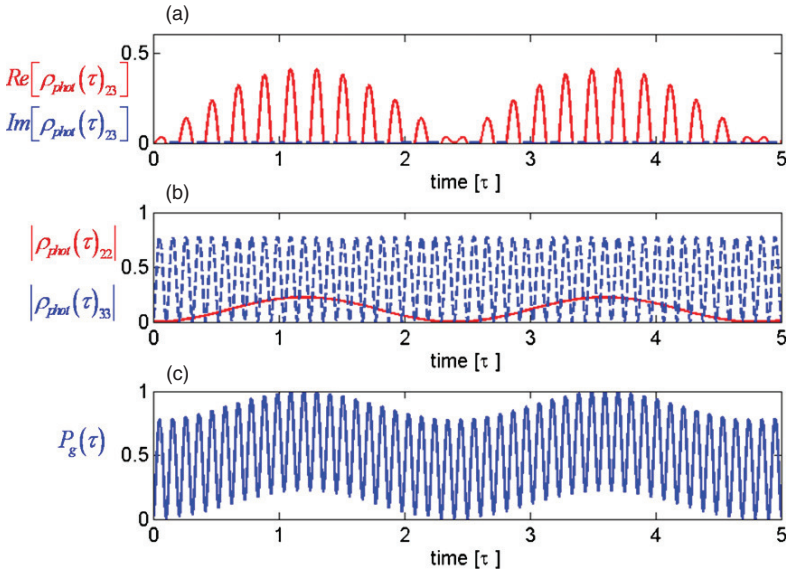

Figure 3. Time evolution of the photon part of the density matrix for *different* coupling strength of the transition paths, with time in units of $\tau = g_1/h$. (a) Real (solid line) and imaginary (dashed line) parts of the off-diagonal element. (b) Absolute values of the diagonal elements for the ω_1 path (solid line) and the ω_2 path (dashed line). (c) Probability for the population of the atomic ground state.

$$\rho_{\text{phot}}(t) = \begin{pmatrix} |\alpha'|^2\cos^2\left(\frac{t|g_1|}{\hbar}\right) + |\beta'|^2\cos^2\left(\frac{t|g_2|}{\hbar}\right) & 0 & 0 \\ 0 & |\alpha'|^2\sin^2\left(\frac{t|g_1|}{\hbar}\cdot\right) & \eta\sin\left(\frac{t|g_2|}{\hbar}\right)\sin\left(\frac{t|g_1|}{\hbar}\right) \\ 0 & \eta^*\sin\left(\frac{t|g_2|}{\hbar}\cdot\right)\sin\left(\frac{t|g_1|}{\hbar}\right) & |\beta'|^2\cdot\sin^2\left(\frac{t}{\hbar}\cdot|g_2|\right) \end{pmatrix} \quad (8)$$

with $\eta = g_2 g_1^* \alpha'^* \beta'/|g_1 g_2|$. The purity of the emitted photon at continuous time t can be assessed by applying an atomic ground-state projection operator $P_i = (|2\rangle\langle 2|) \otimes I_{\text{phot}}$ to the total density matrix, which should, with finite probability, project it onto a one-photon state with two energy components $\widetilde{\rho} = P_i \rho P / tr(P_i \rho)$. The photon-reduced matrix after the projection is

$$\widetilde{\rho}_{\text{phot}}(t) = \mu \begin{pmatrix} 0 & 0 & 0 \\ 0 & |\alpha'|^2 \sin^2\left(\frac{t}{\hbar}|g_1|\right) & \eta^* \sin\left(\frac{t|g_2|}{\hbar}\right)\sin\left(\frac{t|g_1|}{\hbar}\right) \\ 0 & \eta \sin\left(\frac{t}{\hbar}|g_2|\right)\sin\left(\frac{t|g_1|}{\hbar}\right) & |\beta'|^2 \sin^2\left(\frac{t|g_2|}{\hbar}\right) \end{pmatrix} \tag{9}$$

with $\mu = \left[|\alpha'|^2\left(\sin\left(\frac{t}{\hbar}|g_1|\right)\right)^2 + |\beta'|^2\left(\sin\left(\frac{t}{\hbar}|g_2|\right)\right)^2\right]^{-1}$.

The emitted photon is in a pure state regardless of the emission time having $Tr\left(\widetilde{\rho}^2_{\text{phot}}(t)\right) = 1$; however, the amplitudes of the two energy components of the photon are changed by the relative magnitude:

$$\begin{aligned} \Delta\alpha(t) &= \left|\sin\left(\frac{t}{\hbar}|g_1|\right)\sqrt{\mu} - 1\right|^2 \\ \Delta\beta(t) &= \left|\sin\left(\frac{t}{\hbar}|g_2|\right)\sqrt{\mu} - 1\right|^2 \end{aligned} \tag{10}$$

where $\Delta\alpha(t) \triangleq |\alpha'(t) - \alpha'(0)|^2/|\alpha'(0)|^2$, $\Delta\beta(t) \triangleq |\beta'(t) - \beta'(0)|^2/|\beta'(0)|^2$, and $t = 0$ is defined as the time when the electron is in the ground state with unity probability.

An additional physical mechanism, that may deteriorate the purity of the emitted state, is decoherence. It may be included into density matrix formalism [7]:

$$\begin{aligned} &\mathrm{d}\rho/\mathrm{d}t = -\mathrm{i}/\hbar\,[H, \rho] - \widetilde{\Gamma}, \\ &\widetilde{\Gamma}_{ij} = \gamma_d \rho_{ij} \left[\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_{\text{atom}} \otimes I_{\text{photon}}\right] \end{aligned} \tag{11}$$

where γ_d is the electron decoherence rate.

The experimental implementation of the proposal can be quite involved; however, in order to estimate the feasibility of the proposed scheme in principle, we consider a possible practical realization of the system by Rb gas cell, using the states $|5S_{1/2}, F = 1\rangle$, $|5P_{1/2}, F = 2\rangle$, $|5P_{3/2}, F = 2\rangle$ as the states $|2\rangle$, $|1\rangle$, $|3\rangle$ respectively in our model [8], with transition energies $E_{1\to2} = h \cdot 3.77 \cdot 10^{14} Hz$, $E_{3\to2} = h \cdot 3.84 \cdot 10^{14} Hz$ [9, 10]. For a decoherence rate of $\gamma_d \sim 1$ MHz (worst-case scenario) [11, 12] assuming the system's decoherence is determined by the hyperfine splitting of the ground level, coupling energies $g_1 \sim g_3 \sim h \cdot 0.1$ GHz and typical photon lifetime in a single-atom laser cavity [13] of $\sim$1 μs. The long Rabi oscillation period allows us to switch open the cavity at the right time with a negligible timing error (figure 2). Several methods for optical switching at rates of $\sim$10 GHz and higher have been reported [14, 15], enabling handling a process with a period of the order $\sim$10 ns with a timing precision of $\sim$1% of the cycle. According to equation (11) with the parameters of the Rb, the estimated relative error in the qubit coefficients is as low as $\Delta\alpha(\Delta t) \sim \Delta\beta(\Delta t) \sim 10^{-4}$ for a switching time error of $\Delta t = 0.1$ ns. This small error is attributed to the fact that the amplitudes change slowly near the ideal coupling-out time ($t = 0$). The fidelity of the emitted photon state is generally affected by the fidelity of the STIRAP. Several parameters can deteriorate the fidelity such as laser detuning and pulse delay [16]. Theoretical limits on the infidelity for optimized pulse shapes were shown to reach values much lower than those of typical Gaussian pulses $\sim 10^{-4}$ [17], while the highest reported experimental fidelity results are $\sim$95%. [18]. Although in the microwave range, the thermally induced photon noise is significant [19] even at very low temperatures, in the visible range relevant to our scheme the thermal photon noise is estimated at room temperature to be less than 10^{-10} photons, and it can therefore be neglected.

In conclusion, we have proposed and analyzed a scheme for generating arbitrary pure-sate photon-energy qubits, based on adiabatic manipulations of a V-type atom–photon system in a cavity. Our analysis including system dynamics and decoherence reveals several advantages of employing an equal coupling constant V-type system such as robustness and flexibility. The proposed system appears to be realizable in Rb-based configurations.

References

[1] Gisin N, Ribordy G, Tittel W and Zbinden H 2002 Quantum cryptography *Rev. Mod. Phys.* **74** 145

[2] Hayat A, Ginzburg P and Orenstein M 2007 High-rate entanglement source via two-photon emission from semiconductor quantum wells *Phys. Rev.* B **76** 035339

[3] Hayat A, Ginzburg P, Neiman D, Rosenblum S and Orenstein M 2008 Hyperentanglement source by intersubband two-photon emission from semiconductor quantum wells *Opt. Lett.* **33** 1168–70
Hayat A, Ginzburg P and Orenstein M 2009 Photon energy entanglement characterization by electronic transition interference *Opt. Express* **17** 21280

[4] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 Periodic spontaneous collapse and revival in a simple quantum model *Phys. Rev. Lett.* **44** 1323

[5] Bergmann K, Theuer H and Shore B W 1998 Coherent population transfer among quantum states of atoms and molecules *Rev. Mod. Phys.* **70** 1003–25

[6] Jaynes E T and Cummings F W 1963 Comparison of quantum and semiclassical radiation theories with application to the beam maser *Proc. IEEE* **51** 89–109

[7] Scully M O and Zubairy M S 2002 *Quantum Optics* (Cambridge: Cambridge University Press)

[8] de Echaniz S R, Greentree A D, Durrant A V, Segal D M, Marangos J P and Vaccaro J A 2001 Observations of a doubly driven V system probed to a fourth level in laser-cooled rubidium *Phys. Rev.* A **64** 013812

[9] Barwood G P, Gill P and Rowley W R C 1991 Frequency measurements on optically narrowed Rb-stabilised laser diodes at 780 nm and 795 nm *Appl. Phys.* B **53** 142

[10] Ye J, Swartz S, Jungner P and Hall J L 1996 Hyperfine structure and absolute frequency of the ^{87}Rb $^5P_{3/2}$ state *Opt. Lett.* **21** 1280

[11] Raitzsch U, Heidemann R, Weimer H, Butscher B, Kollmann P, Löw R, Büchler H P and Pfau T 2009 Investigation of dephasing rates in an interacting Rydberg gas *New J. Phys.* **11** 055014

[12] Sautenkov V A, Rostovtsev Y V and Scully M O 2005 Switching between photon–photon correlations and Raman anticorrelations in a coherently prepared Rb vapor *Phys. Rev.* A **72** 065801

[13] McKeever J, Boca A, Boozer A D, Buck J R and Kimble H J 2003 Experimental realization of a one-atom laser in the regime of strong coupling *Nature* **425** 268–71

[14] Soljačić M, Lidorikis E, Joannopoulos J D and Hau L V 2005 Ultralow-power all-optical switching *Appl. Phys. Lett.* **86** 171101

[15] Notomi M and Taniyama H 2008 On-demand ultrahigh-Q cavity formation and photon pinning via dynamic waveguide tuning *Opt. Express* **16** 18657–66

[16] Vitanov N V, Suominen K-A and Shore B W 1999 *J. Phys. B: At. Mol. Opt. Phys.* **32** 4535

[17] Vasilev G S, Kuhn1 A and Vitanov N V 2009 Optimum pulse shapes for stimulated Raman adiabatic passage *Phys. Rev.* A **80** 013417

[18] Khudaverdyan M, Alt W, Dotsenko I, Förster L, Kuhr S, Meschede D, Miroshnychenko Y, Schrader D and Rauschenbeutel A 2005 Adiabatic quantum state manipulation of single trapped atoms *Phys. Rev.* A **71** 031404(R)

[19] Gleyzes S, Kuhr S, Guerlin C, Bernu J, Deléglise S, Hoff U B, Brune M, Raimond J-M and Haroche S 2007 Quantum jumps of light recording the birth and death of a photon in a cavity *Nature* **446** 297